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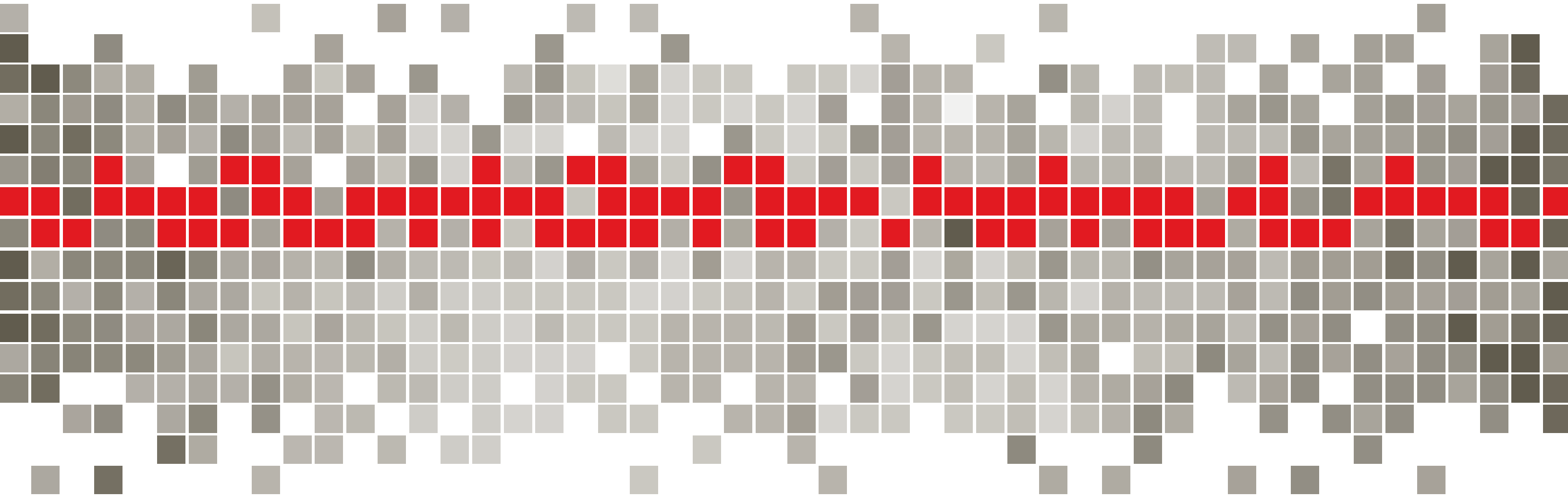
Understanding Inertia Ratio and Its Effect On Machine Performance

White Paper



Classic rules of thumb don't take into account many system characteristics; for best results, only an analytical approach will do.

by Bryan Knight

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TAKEAWAYS:

- High load-to-motor inertia ratios push resonance and anti-resonance peaks to lower frequencies, shrinking the operating bandwidth of the machine.
- Gearboxes can reduce the inertia ratio,enabling higher-speed operation and/or the use of smaller, cheaper motors.
- Modern, high-performance servo drive control and filter algorithms can compensate for the resonance effects better than older technologies, allowing designers to use higher-ratio systems.

INTRODUCTION

Sizing a motor is one of the most fundamental tasks in motion system design. It's also the task most frequently performed incorrectly. The job of the motor is to move the load, so it's easy to make the mistake of selecting it purely on whether it has sufficient torque and speed to match the application. That is an oversimplification that can introduce problems, however. The motor has to do more than simply move the load; it needs to position it how and when desired. Unfortunately, the interaction between the mechanism and the load can introduce resonances, leading to errors like overshoot and extended settling times. In this white paper, we will discuss how the ratio of load inertia to motor inertia affects performance, how additional factors such as compliance come into play and how proper sizing, combined with auto tuning, can help deliver the best possible results.

No one sets out to build a bad machine, but all too frequently a wide gap develops between the desired performance and that of the actual equipment. Sizing a motor based only on torque and speed requirements works fine if the machine has an infinitely stiff mechanism. Unfortunately, mechanical components like shafts, couplings, belts, ballscrews and the machine frame all feature a certain level of compliance. When the motor tries to move the load, the interaction among the applied force, the load inertia and the mechanical compliance excites vibrations in the machine at characteristic resonant frequencies. These vibrations can lead to overshoot, ringing, and sluggish motion that prevent the load from arriving at the commanded location and speed. This becomes even more critical when you are synchronizing multiple axes, as each axis must continuously be at the commanded speed and position at the same time as the other axes. When this synchronization is not maintained at the load, print quality suffers, gantry systems jam, and the machine in general does not perform as designed.

When choosing a motor, design teams frequently resort to rules of thumb that have evolved over time. These approximations are based on a certain set of assumptions that can vary from organization to organization and product to product. The problem is that using these approximations without fully understanding the underlying assumptions can lead to exactly the kinds of issues we mentioned above. Bumping up motor size by some set amount only compounds the error since it doesn't actually address the real problem. At best, you wind up paying for a bigger motor than you need; at worst, you aggravate an existing issue. As with anything in engineering, you can't fix problem until you understand the problem.

INERTIA BASICS

Let's start with a basic motion system of a motor connected to a load via a shaft and coupling (see figure 1). We can approximate the inertia of the motor J_M as the inertia of the rotor, encoder and optional brake (for purposes of this paper, we will assume a classic rotary motor with a turning rotor and stationary stator). The load inertia J_L encompasses everything else – shaft, coupling and load. Now, we can define an inertia ratio as:

$$\text{Inertia ratio} = J_L/J_M$$

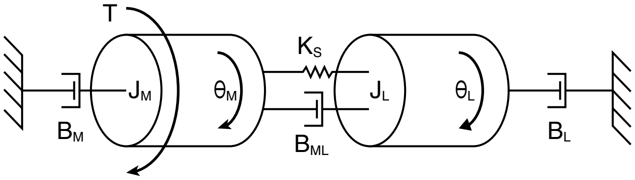

Fig. 1, A basic motion system of a motor with moment of inertia J_M connected to a load with moment of inertia J_L via a shaft with spring constant K_s and viscous damping B_M, B_{ML}, and B_L.

It's important to note that the load inertia includes more than just the tooling or product you are moving. Gearheads and actuators also contribute, along with their couplings. Depending on the system, the load inertia can encompass contributions made by multiple layers of gear reductions and actuators. At this point, it's useful to consider everything beyond the rotor to be a black box with some reflected inertia J_L "seen" by the rotor.

We can model the compliance as a spring with spring constant Ks (see figure 1). When the rotor turns, the inertia of the load winds up the spring, the higher the load inertia, the greater the wind up. This compliance degrades the ability of the axis to position the load as commanded. When the wind/unwind motion is in phase with the motion of the motor, it adds to the desired motion and can lead to oscillation or instability. This occurs at the resonant frequency of the machine. This resonance can be mechanically damped to reduce its effect. Alternatively, the resonant frequency can be increased by stiffening the spring constant of the mechanism or by reducing the inertia ratio JM/JL. Because higher resonance frequencies are easier to filter electronically, choosing a high-quality coupling and reducing the inertia ratio is an ideal method of improving performance.

In contrast, when the wind/unwind motion is out of phase with the motion of the motor, it cancels the some or all of the desired motion of the motor. This leads to sluggish performance where the load significantly lags the commanded position. In extreme cases of reciprocating motion, the load may not move at all.

To get a better understanding of the effect, let's look at the Bode plot of gain (see figure 2). For ideal performance, we want our system to deliver a consistent response across the effective range of operating frequencies. The curves show a decrease in gain with increasing frequency but also reveal anti-resonance valleys and resonance peaks. As the speed of a machine increases, it will go from performing as expected to suddenly performing very sluggishly (anti-resonance) to wildly over responding (resonance). At best, this behavior might prevent your machine from positioning as desired. At worst, it could cause jams, damage product, wreck equipment, or even harm personnel.

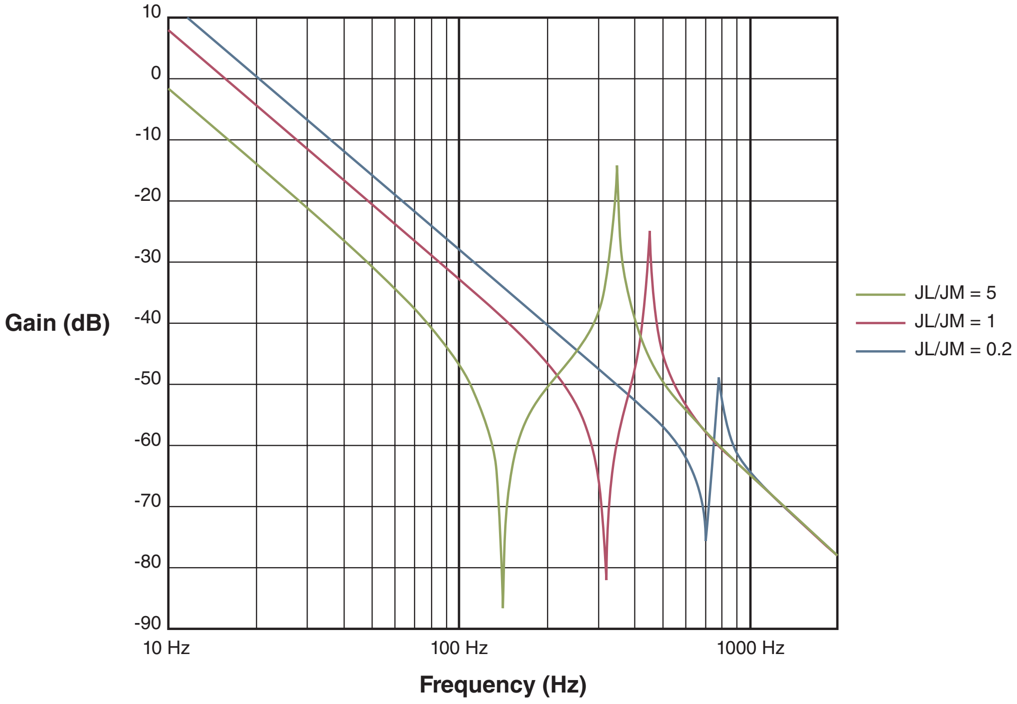

Fig. 2, Plot of gain versus frequency reveals resonance and anti-resonance points for varying inertia ratios. Notice how raising the inertia ratio increases the magnitude and bandwidth of the unstable region while reducing the frequency.

If we had a smooth, flat curve, we could increase the gain of the servo drive. A quick glance at the figure shows that high gain would lead to catastrophic behavior around the resonance peak, however. An obvious solution would be to filter the motion to decrease the gain at the resonance peaks, then increase the overall gain to push the curve upward. This is the point at which inertia matching becomes important.

As the figure shows, the higher the inertia ratio, the larger the peaks and the greater the bandwidth they occupy. More importantly, higher inertia ratios push the resonance and anti-resonance peaks down to lower frequencies, significantly curtailing the effective operating bandwidth of the system. Your motor might have enough torque to move the load, but it may not be able to do it quickly and smoothly enough to satisfy the application.

In this context, the dangers of simply using a rule of thumb to size a motor become apparent. Putting together a machine with an excessively high inertia ratio for the operating conditions and application is one of the factors responsible for the gap between required and actual performance. Motor size needs to be determined using not just speed and torque requirements but within the context of the overall design. How fast does the axis need to move? How much space do you have? What's the budget? The design isn't complete until it best balances all the requirements. Fortunately, engineers have multiple degrees of freedom they can apply to solve the problem.

Inertia ratio from a systems perspective

The rules of thumb for inertia ratio evolved as experience-based shortcuts, but they should not be applied across the board. A pick-and-place gantry loading products in a case packer, for example, could be built with an inertia ratio of 5:1 or even 10:1 and still meet the specifications for settling time and smoothness. A wafer-handling machine in an IC fab might need an inertia ratio of closer to 1:1, given that requirements for settling time, smoothness and positioning accuracy are much more stringent. Higher quality components with high stiffness do not change the overshoot and settling time requirements for an application, but these components can alter the resonant frequency, allowing the designer greater flexibility in selecting a servo motor.

The same is true in rotary applications. For example, a press feed with considerable friction in the system may work just fine with a much higher inertia than that of a printing press, for which the phase errors between axes must be nearly zero and the high-precision components of the system offer little friction to dampen resonance effects. By understanding both your requirements and machine components and tolerances, you can predict and design for the performance your application requires.

When you are investigating servo motors and drives from different manufacturers, it can be tempting to just try to cross over from one motor to another. That is a technique that can backfire, though. Breakpoints of motor families rarely correlate between suppliers. The components of each vendor feature slightly different parameters and performance, so a rule of thumb that may work well with one system's unique set of filters, feedback resolution and internal speed frequency response may not work with another system's unique specifications. Inertia ratio should be considered on a case-by-case basis.

The common practice of starting with the motor from a previous platform and adding a 5% or 10% safety margin relies more on luck than science, and doesn't deliver a cost-efficient solution. Not only is the motor bigger, more power-hungry and more expensive, it requires a bigger drive. The approach also compounds the problem each generation because the multiplier gets applied to a motor that was already oversized in the first place.

Alternatively, designers may err in the other direction by choosing a smaller motor, perhaps to cut costs or reduce part numbers. This can backfire in a big way. You might save on the initial capital expenditure but you run the risk of increasing inertia ratio and pushing resonance frequencies into your operating bandwidth. Oversizing or undersizing the motor and components rarely leads to optimum operation. In some cases, the penalty is going over budget, but most of the time, the penalty is sub-par machine performance. It's better to begin with your specifications and choose a fresh motor from there.

Gearboxes – the "magic bullet"

Increasing motor size is not the only way or even the best way to improve the inertia ratio. Reducing the inertia of the load by decreasing the mass is the best approach to try for performance gains. Once the unnecessary fat has been trimmed, gearboxes are the tool of choice to modify the inertia ratio without increasing motor size. A gearbox converts excess speed capability into torque, which makes it possible to use a smaller motor for the same load. Based on our previous discussion, the inertia of the gearbox adds to J_L. We would expect that to increase the inertia ratio, which would worsen resonance issues, but that is not the case.

For a gearbox with gear ratio N attached to a motor, the output torque τ scales linearly as N while the angular velocity ω goes inversely:

$$\tau_{\phi\,=}\tau_{\iota}/N \qquad [2]$$

$$\omega_{\phi\,=}\omega_{\iota}/N \qquad [3]$$

The real benefit of the gearbox though is that it greatly reduces the reflected load inertia seen by the motor:

$$J_R = J_L/N^2$$

In other words, adding a gearbox scales the reflected inertia by the inverse square of the reduction at each step. This more than makes up for the smaller motor and the added inertia of the gearbox, making the gearbox the "magic bullet" of motor sizing. Simply switching from a 5:1 gearbox to 6:1 gearbox and using the full speed capability of the motor, for example, can improve inertia ratio by 44% and torque by 20%. If this combination yields excess torque capability and meets the speed requirements, it enables the use of a smaller motor.

As always, there are trade-offs. This approach saves money in motor costs but the gearbox adds cost and, potentially, maintenance (unless you choose a "lubricated for life" device). Other benefits of a smaller motor coupled with a gearbox include lower energy consumption, reduced electrical enclosure size and greater mechanical design flexibility.

We gave a simple example above, but reflected inertia can be broken into multiple levels with inertia scaling according to equation 4 at each step. Instead of a simple gearbox and load, an axis might include a gearbox driving a belt reducer and then the load. At each point, the mechanical components not only introduce a mechanical advantage in torque, they also further scale the reflected inertia.

Machine design needs to be considered from a systems point of view. Adding a gear or belt reduction might modify reflected inertia but more components also introduce more compliance, which can counteract the benefits. In addition, they add cost, size and points of failure. Both integration and maintenance become more complex. Choose components based on an understanding of how each component affects performance.

MANAGING INERTIA

Once the mechanical design has been optimized, the next step is to enhance performance using the electromechanical properties of the system. Ideally, we want flat gain so that the machine will respond consistently across all speeds. As figure 2 shows, however, gain typically falls off with increasing frequency. We could turn up the gain to make the system more responsive, but across the frequency band around the anti-resonance and resonance peaks, system behavior will be erratic, at best. The solution is to maximize performance using the control loop tuning and filtering functions of the servo drive.

There was a time that choosing and installing a motor was only half the battle. To make an axis perform properly, engineers had to painstakingly tune the output signal of the servo drive by adjusting potentiometers that controlled the proportional, integral and derivative (PID) gains. This PID loop tuning technique was a process that could take hours, if not days. Today, high-functionality drives like the Mitsubishi Electric MR-J4 apply system feedback to tune themselves automatically, leveraging multiple types of filters to compensate for resonances and flatten gain.

Fig. 3, The MR-J4 auto tuning drive applies proactive tuning to avoid exciting anti-resonance and resonance spikes, and reactive auto tuning to optimize performance in real time

The MR-J4 features proactive and reactive tuning capabilities that combine to optimize performance. For proactive tuning, the drive begins with a pair of Vibration Suppression Control filters. These filters are designed to reduce or remove the resonance peaks resulting from static vibration modes inherent to the combination of the mechanism and the static load inertia.

The reactive auto tuning process starts with observing the behavior of an axis using the actual command sequence from the application. The drive compares the results to those from an ideal virtual "observer" system. Based on the deviation of the real system from the ideal, the drive characterizes the mechanical system, including determining the load inertia ratio, and optimizes the control loop and filter parameters based on these system characteristics. The optimized loop and filter settings adjust the subsequent command signals in order to compensate for the effects of resonances to maximize responsiveness and minimize error. As system characteristics such as load inertia change, whether suddenly by picking up a part or slowly during the unwinding of a roll of material, the system continually determines the optimal settings to ensure consistent performance over a wide range of operating conditions.

Of course, no machine remains unchanged over time. Bearings wear, belts stretch, end-users increase speeds or change product size and weight. Autotuning enables the machine to maintain its performance in the face of continual small modifications.

ADDING IT ALL UP

Rules of thumb evolved with time and experience to simplify the process of motor sizing, especially prior to the availability of mechatronic design tools. Although there are no hard and fast rules, based on the analysis above, we can make some generalizations. Obviously, compliance is bad when it comes to response, but removing all system compliance is likely cost-prohibitive. In these cases, the system benefits greatly from any effort to lower inertia ratio. The opposite is also true,a very stiff machine can deliver good performance even with a high inertia ratio.

Engineering is about trade-offs.

There is no perfect solution, just the best solution for your application. You could build an incredibly stiff machine for optimal performance, but the cost difference between a part with a tolerance of 1.00 inches and 1.0000 inches can amount to thousands of dollars. If you're building a highly accurate 20-color printing press, details like axis-to-axis performance consistency, alignment, coupling types, shaft design and so on become extremely important. Tight tolerances on these components are essential. On the other hand, an application like case packing or palletizing can easily tolerate longer settling time or slight overshoot. In this case, a design with a large amount of flexibility and higher inertia ratio is not a problem.

Ultimately, it comes down to the requirements: How accurate do you need your positioning and synchronization to be? How much are you (or your customer) willing to spend to stiffen up the machine and reduce mass? You can use rules of thumb and compensate by applying much of your budget to mechanical components or you can properly size your motor and add a gearbox to get much better performance at a more reasonable price. Add to that the tuning and compensation capabilities offered by cutting-edge drives like the MR-J4 and you can easily develop a cost-effective system that will deliver the exact response you need.

Further reading

Autotuning Servos Optimize Machine Performance Effortlessly

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